X Junior Balkan Olympiad in Informatics

Skopje, 2016

Day 1: Party of programmers

Short description:

The pairs of friends defines an acyclic graph G(V, E).

The solution is the following:

Let's color black the vertices which correspond to the people who accepted the initial invitation, and let's define

- max: the number of programmers who accepted the initial invitation.

- m: the number of programmers with no friends who accepted the initial invitation

Then perform a BFS with starting vertices those which correspond to the programmers who accepted the initial invitation. Then, in each step of the BFS we do the following

- If a programmer is colored black has an uncolored friend, color its friend white.
- If a programmer is colored white has an uncolored friend, color its friend black.
- If a programmer is colored black has a friend colored white, do nothing.
- If a programmer is colored white has a friend colored black, do nothing.
- If a programmer is colored black has a friend colored black as well, change the color of the later to red.
- If a programmer is colored white has a friend colored white as well, change the color of the later to red.
- All friends of a programmer colored red must be colored red as well.

When the BFS terminates, compute the number of vertices colored black, n1, the number of vertices colored white, n2 and the number of vertices colored red, n3.

The final answer will be: max{max, n1+n3-m, n2+n3-m}

Analysis:

Let A be the set of all programmers who accepted the initial invitation. Note if a programmer has no friends may accept the invitation if and only if he accepted initially and never after that.

In a connected component, if there are programmers in the set A who are connected with a path of odd length, or there is odd length cycle which contains a vertex in A, then after a number of steps two friends will accept the invitation in the same step. After that, the status of those two will never change. Moreover, after a number of steps all programmers of their connected component will accept the invitation, and that situation will never change after that.

Also, in a connected component, if there are no programmers from A who are connected with an odd length path, and all cycles containing a vertex in A have an even length, then after a number of steps all programmers in an even distance from the programmers in A will be accept, and in the next step those who did not accept previously, will accept and those who previously accepted will reject the invitation. In each step after that the status of the graph will be changing alternatively, the programmers accepting in the i_{th} step reject the invitation in the $i+1_{th}$, and the programmers rejecting in the i_{th} step accept in the $i+1_{th}$ step.

Since it is possible that the set of programmers who accepted the initial invitation is maximal, we need check the initial situation as well.